Transforming function

$$A\,f(B(x+C))+D$$

A: Vertical stretch

B: Horizontal stretch

C: Horizontal translation

D: Vertical translation

A

A f(x)

A multiplies the y-coordinates.

The effect of **A** is to **vertically stretch** the graph by the scale factor **A**.

1. (a) Sketch the graph of f(x) = x^2.

(b) On the same set of axes, sketch the graph of

(i) 3 f(x)

(ii) $\frac{1}{2}$ f(x)

2. (a) Sketch the graph of f(x) = e^x.

(b) On the same set of axes, sketch the graph of

(i) 5 f(x)

(ii) $\frac{1}{4}$ f(x)

B

f(Bx)

The x-coordinates are divided by **B**.

The effect of **B** is to **horizontally stretch** the graph by the scale factor $\frac{\mathbf{1}}{\mathbf{B}}$.

1. (a) Sketch the graph of f(x) = x^2.

(b) On the same set of axes, sketch the graph of

(i) f(2x)

(ii) f($\frac{1}{2}$x)

2. (a) Sketch the graph of f(x) = ln x.

(b) On the same set of axes, sketch the graph of

(i) f(4x)

(ii) $f(\frac{1}{3}x)$

C

f(x + C)

The effect of **C** is to **horizontally translate** the graph through **C** units.
If C is Positive, shift to Left
If C is Negative, shift to Right

1. (a) Sketch the graph of f(x) = x^2.
(b) On the same set of axes, sketch the graph of
(i) f(x – 3)
(ii) f(x + 1)

2. (a) Sketch the graph of f(x) = e^x.

(b) On the same set of axes, sketch the graph of

(i) f(x + 4)

(ii) f(x – 2)

D

f(x) + D

The effect of **D** is to **vertically translate** the graph through **D** units.
If D is Positive, shift Up
If D is Negative, shift Down

1. (a) Sketch the graph of f(x) = x^2.
(b) On the same set of axes, sketch the graph of
(i) f(x) – 3
(ii) f(x) + 2

2. (a) Sketch the graph of f(x) = e^x.

(b) On the same set of axes, sketch the graph of

(i) f(x) + 2

(ii) f(x) – 1

Exercise
Paper 1

1. Let $f(x) = 2\ln x$ and $g(x) = \ln 5x^2$.

(a) Express $g(x)$ in the form $f(x) + \ln a$, where $a \in \mathbb{Z}^+$.
(b) The graph of g is a transformation of the graph of f. Give a full geometric description of this transformation.

2. Let f(x) = x^2 + 4 and g(x) = x – 1.

(a) Find $(f \circ g)(x)$.

The vector $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ translates the graph of $(f \circ g)$ to the graph of h.

(b) Find the coordinates of the vertex of the graph of h.

(c) Show that h(x) = x^2 – 8x + 19.

(d) The line y = 2x – 6 is a tangent to the graph of h at the point P. Find the x-coordinate of P.

Paper 2

1. Let $f(t) = 2t^2 + 7$, where $t > 0$. The function v is obtained when the graph of f is transformed by a stretch by a scale factor of $\frac{1}{3}$ parallel to the y-axis, followed by a translation by the vector $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$.

(a) Find $v(t)$, giving your answer in the form $a(t-b)^2 + c$.

(b) A particle moves along a straight line so that its velocity in ms^{-1}, at time t seconds, is given by v. Find the distance the particle travels between $t = 5.9$ and $t = 6.8$.

2. Let $f(x) = 3x^2$. The graph of f is translated 1 unit to the right and 2 units down.
The graph of g is the image of the graph of f after this translation.
(a) Write down the coordinates of the vertex of the graph of g.
(b) Express g in the form $g(x) = 3(x - p)^2 + q$.

The graph of h is the reflection of the graph of g in the x-axis.
(c) Write down the coordinates of the vertex of the graph of h.