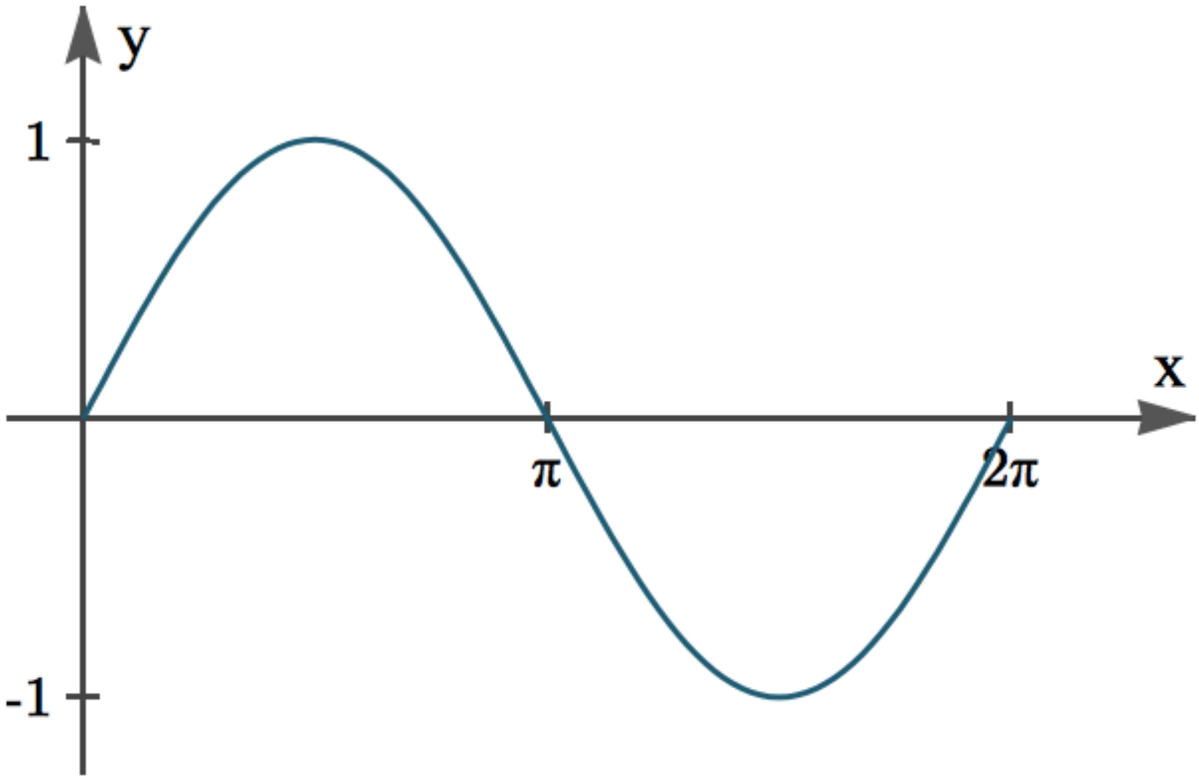
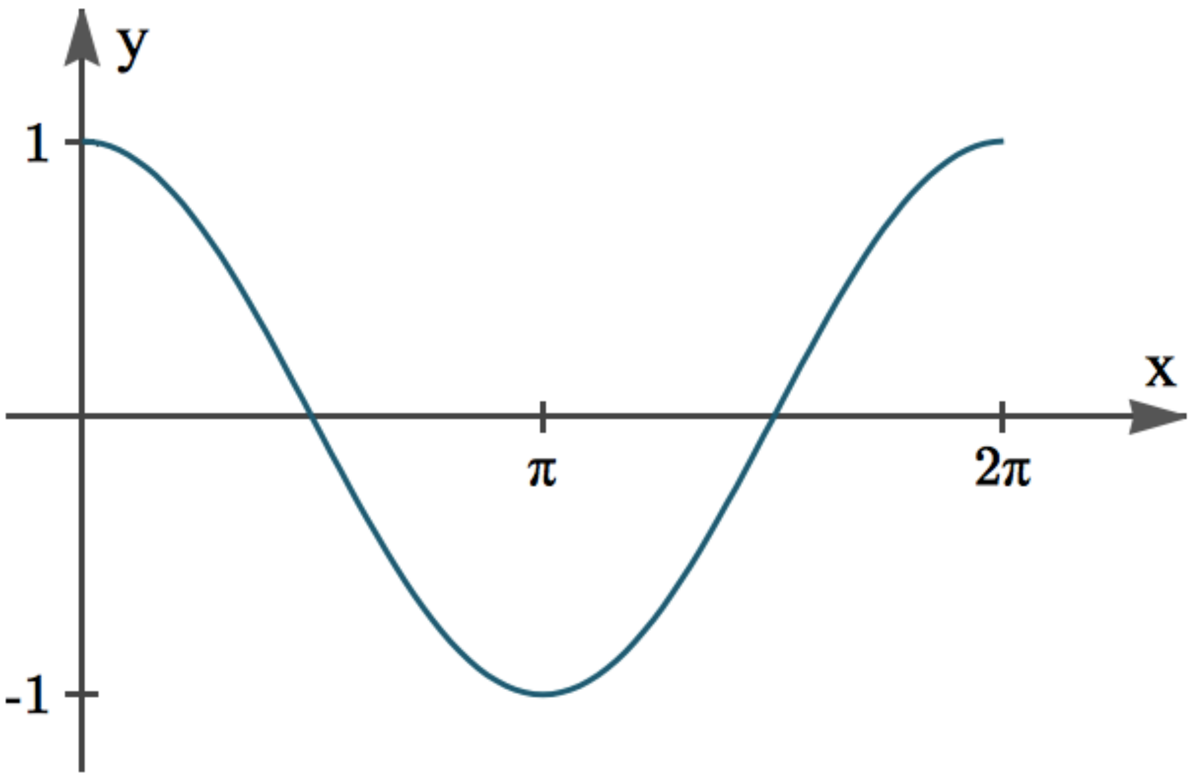
Trigonometric transformation

Sine graph



Cosine graph



Transformation

$$A\,f(B(x+C))+D$$

A: Affects amplitude
B: Affects period
C: Affects horizontal translation
D: Affects vertical translation

The **period** for sine and cosine function is 2π.
Amplitude is half of the vertical distance.

A

A f(x)

A is amplitude.

A multiplies the y-coordinates.

A = $\frac{y_{max} - y_{min}}{2}$

1. Sketch the graph of the followings.

(a) y = 2 sin x

(b) y = 0.5 sin x

B

f(Bx)

B affects the period.

New period = $\boldsymbol{\frac{2\pi}{B}}$

1. Sketch the graph of the followings.

(a) sin 2x

(b) sin $\frac{3}{2}x$

C

f(x + C)

C is the horizontal shift.

If C is **Positive**, shift **Left**
e.g. f(x + 2) means shift Left by 2.

It C is **Negative**, shift **Right**
e.g. f(x – 1) means shift Right by 1.

1. Sketch the graph of the followings.
(a) sin (x + $50°$)

(b) sin (x – π)

D

f(x) + D

D is the vertical shift.

If **D** is Positive, shift **Up**
e.g. f(x) + 1 means shift Up by 1.

It **D** is Negative, shift **Down**
e.g. f(x) – 3 means shift Down by 3.

1. Sketch the graph of the followings.
(a) sin x + 2

(b) sin x – 1

Exercise
Paper 1

1. $f(x) = \cos 2x$ and $g(x) = 2x^2 - 1$.

(a) Find $f\left(\frac{\pi}{2}\right)$.

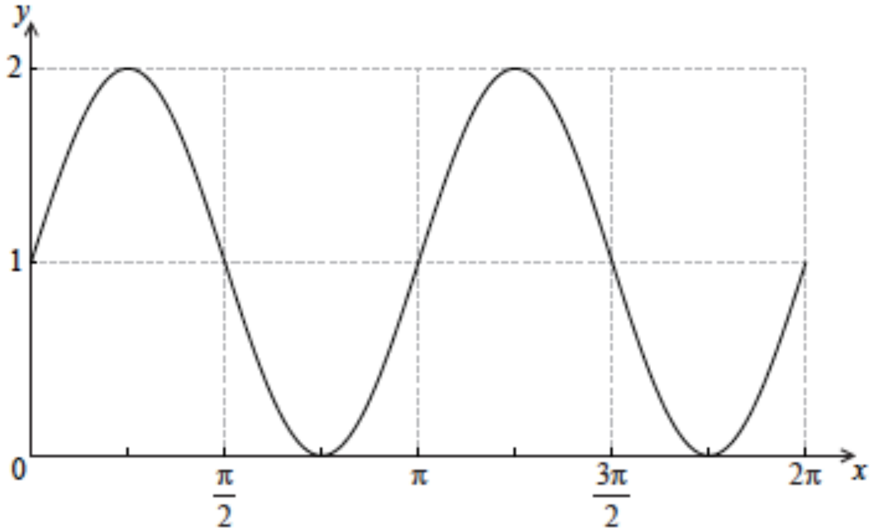
(b) Find $(g \circ f)\left(\frac{\pi}{2}\right)$.

(c) Given that $(g \circ f)(x)$ can be written $a \cos(kx)$, find the value of k, $k \in \mathbb{Z}$.

2. Let $f(x) = (\sin x + \cos x)^2$.

(a) Show that $f(x)$ can be expressed as $1 + \sin 2x$.

The graph of f is shown below for $0 \leq x \leq 2\pi$.



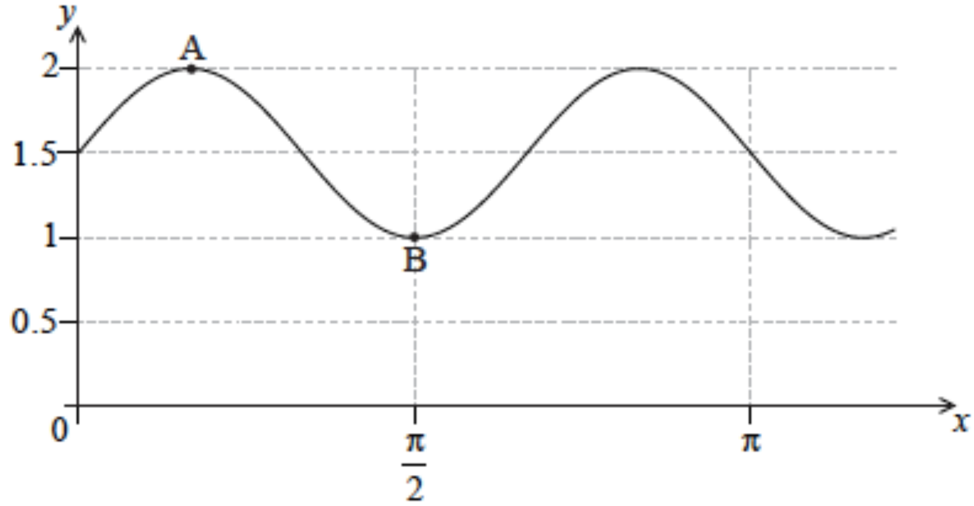
(b) Let $g(x) = 1 + \cos x$. On the same set of axes, sketch the graph of g for $0 \leq x \leq 2\pi$.

The graph of g can be obtained from the graph of f under a horizontal stretch of scale factor p followed by a translation by the vector $\begin{pmatrix} k \\ 0 \end{pmatrix}$.

(c) Write down the value of p and a possible of k.

Paper 2

1. The following diagram shows part of the graph of *y=p sin(qx)+r.*



The point A$\left(\frac{\pi}{6}, 2\right)$ is a maximum point and the point B$\left(\frac{\pi}{2}, 1\right)$ is a maximum point.
Find the value of

(a) p
(b) r
(c) q

2. The depth of water in a port is modelled by the function

$d(t) = p \cos qt + 7.5$, for $0 \leq t \leq 12$, where t is the number of hours after high tide.

At high tide, the depth is 9.7 metres.
At low tide, which is 7 hours later, the depth is 5.3 metres.

(a) Find the value of p.
(b) Find the value of q.
(c) Use the model to find the depth of the water 10 hours after high tide.