Vector intersection

Just like the regular functions,

1. Equate the two vector equations

2. Solve the parameter "t" or "s" by substitution

3. Put one of them into their vector equation

4. Write the answer as coordinates.

1. Line L_1 has equation $r_1 = \begin{pmatrix} -2 \\ 1 \end{pmatrix} + s\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and line L_2 has equation $r_2 = \begin{pmatrix} 15 \\ 5 \end{pmatrix} + t\begin{pmatrix} -4 \\ 1 \end{pmatrix}$.

Line L_1 and L_2 intersect at point A. Find the coordinates of A.

2. Line L_1 has equation $r_1 = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + s\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$ and line L_2 has equation $r_2 = \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix} + t\begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix}$.

Line L_1 and L_2 intersect at point A. Find the coordinates of A.

A point lies on the vector

1. $L_1: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \end{pmatrix} + t\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

The point $A(7, p)$ lies on L_1.
Find the value of p.

2. $L_1: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix} + t\begin{pmatrix} 7 \\ 3 \\ q \end{pmatrix}$

The point $A(15, p, 17)$ lies on L_1.
Find the value of p and q.

Paper 1

1. The line L passes through the point $(5, -4, 10)$ and is parallel to the vector $\begin{pmatrix} 4 \\ -2 \\ 5 \end{pmatrix}$.

(a) Write down a vector equation for line L.

(b) The line L intersects the x-axis at the point P. Find the x-coordinate of P.

2. A line L_1 passes through points $P(-1, 6, -1)$ and $Q(0, 4, 1)$.

(a) (i) Show that $\overrightarrow{PQ} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$.

(ii) Hence, write down an equation for L_1 in the form $r = a + tb$.

A second line L_2 has equation $r = \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} + s\begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix}$.

(b) Find the cosine of the angle between $\overrightarrow{PQ}$ and L_2.

(c) The lines L_1 and L_2 intersect at the point R. Find the coordinates of R.

Paper 2

1. Line L_1 has equation $r_1 = \begin{pmatrix} 10 \\ 6 \\ -1 \end{pmatrix} + s\begin{pmatrix} 2 \\ -5 \\ -2 \end{pmatrix}$ and line L_2 has equation $r_2 = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} + t\begin{pmatrix} 3 \\ 5 \\ 2 \end{pmatrix}$.

Line L_1 and L_2 intersect at point A. Find the coordinates of A.

2. Line L_1 passes through points $A(1,-1,4)$ and $B(2,-2,5)$.

(a) Find $\overrightarrow{AB}$.

(b) Find an equation for L_1 in the form $r = a + tb$.

Line L_2 has equation $r = \begin{pmatrix} 2 \\ 4 \\ 7 \end{pmatrix} + s\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$.

(c) Find the angle between L_1 and L_2.

(d) The lines L_1 and L_2 intersect at point C. Find the coordinates of C.

3. Consider the line L_1 and L_2 with equations

$$L_1 : r = \begin{pmatrix} 11 \\ 8 \\ 2 \end{pmatrix} + s\begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix} \text{ and } L_2 : r = \begin{pmatrix} 1 \\ 1 \\ -7 \end{pmatrix} + t\begin{pmatrix} 2 \\ 1 \\ 11 \end{pmatrix}.$$

The lines intersect at point P.

(a) Find the coordinates of P.

(b) Show that the lines are perpendicular.

(c) The point $Q(7, 5, 3)$ lies on L_1. The point R is the reflection of Q in the line L_2. Find the coordinates of R.