Integration by substitution

No product rule and quotient rule for integration

Multiply

1. $\int (x^2 + 3x)^3 (2x + 3)\, dx$

Let $u = x^2 + 3x$

$\frac{du}{dx} = 2x + 3$

$dx = \frac{du}{2x+3}$

$\int u^3 (2x + 3)\, dx$

$= \int u^3 (\cancel{2x + 3}) \frac{du}{\cancel{2x+3}}$

$= \int u^3\, du$

$= \frac{1}{4} u^4 + C$

$= \frac{1}{4} (x^2 + 3x)^4 + C$

2. $\int 3x^2(x^3 - 2)\,dx$

Fraction

1. $\int \frac{3x^2+2}{x^3+2x} dx$

Let $u = x^3 + 2x$

$\frac{du}{dx} = 3x^2 + 2$

$dx = \frac{du}{3x^2+2}$

$\int \frac{3x^2+2}{u} dx$

$= \int \frac{\cancel{3x^2+2}}{u} \text{ x } \frac{du}{\cancel{3x^2+2}}$

$= \int \frac{1}{u} du$

$= \ln u + C$

$= \ln x^3 + 2x + C$

2. $\int \frac{4x^3-1}{x^4-x}\,dx$

Paper 1

1. Let $f'(x) = sin^3(2x)\cos(2x)$.

   Find $f(x)$, given that $f\left(\frac{\pi}{4}\right) = 1$.

2. Let $g(x) = \frac{\ln x}{x}$.

(a) Find $g'(x)$.

(b) Find $\int g(x)dx$.