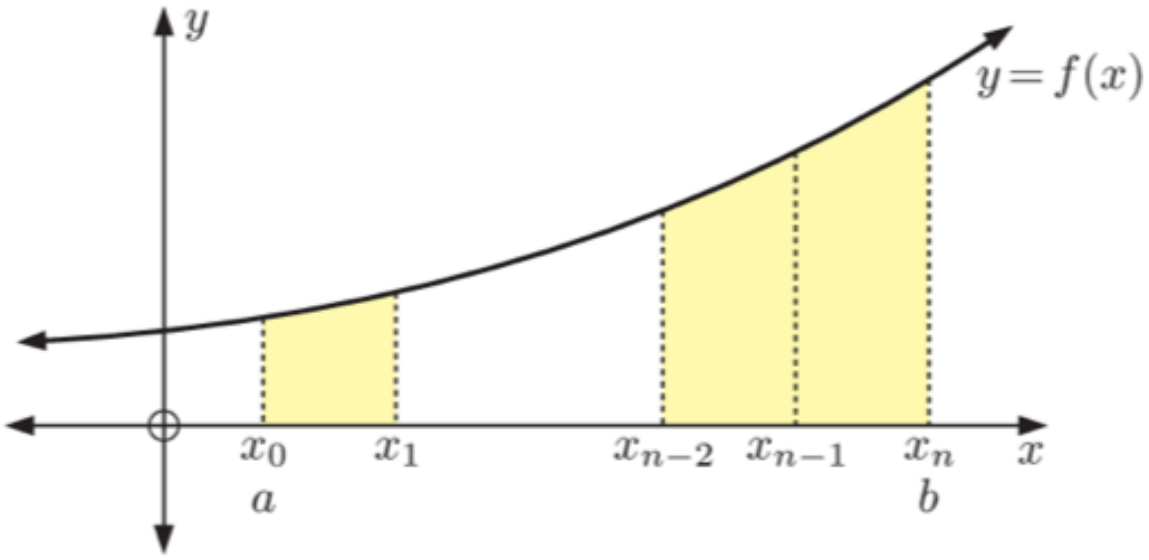
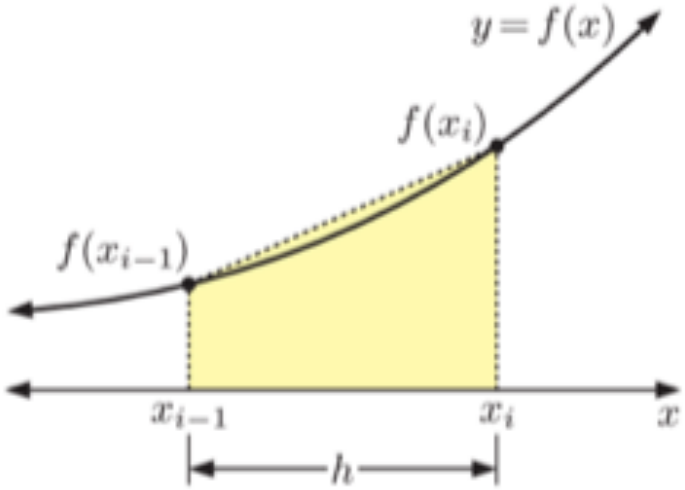
## Trapezoidal rule




$\int_a^b y\,dx \approx \frac{1}{2}h\big((y_0 + y_n) + 2(y_1 + y_2 + \cdots + y_{n-1})\big)$, where $h = \frac{b-a}{n}$

1. Use the trapezoidal rule with 6 subintervals to estimate the area between $f(x) = \sqrt{6 - x^2}$ and the x-axis from $x = 1$ to $x = 2$.

2. Use the trapezoidal rule with 4 subintervals to approximate the area between the x-axis and:

(a) $f(x) = \frac{2}{\sqrt{x}}$ from $x = 2$ to $x = 4$

(b) $f(x) = -x^2 + 6x - 4$ from $x = 1$ to $x = 3$

3. (a) Use the trapezoidal rule with 6 subintervals to calculate the area between the x-axis and $f(x) = 3 - x$ from $x = 0$ to $x = 3$.

## Integration rules

### Reverse of differentiation

Find the derivative of $f(x) = x^3$ and $f(x) = x^3 + 10$.

## Indefinite integral:

$$\int \boldsymbol{f'(x)\,dx = f(x) + C}$$

C is constant.
Derivative of a constant is 0.

$$\int \boldsymbol{x^n\,dx = \frac{x^{n+1}}{n+1} + C}$$

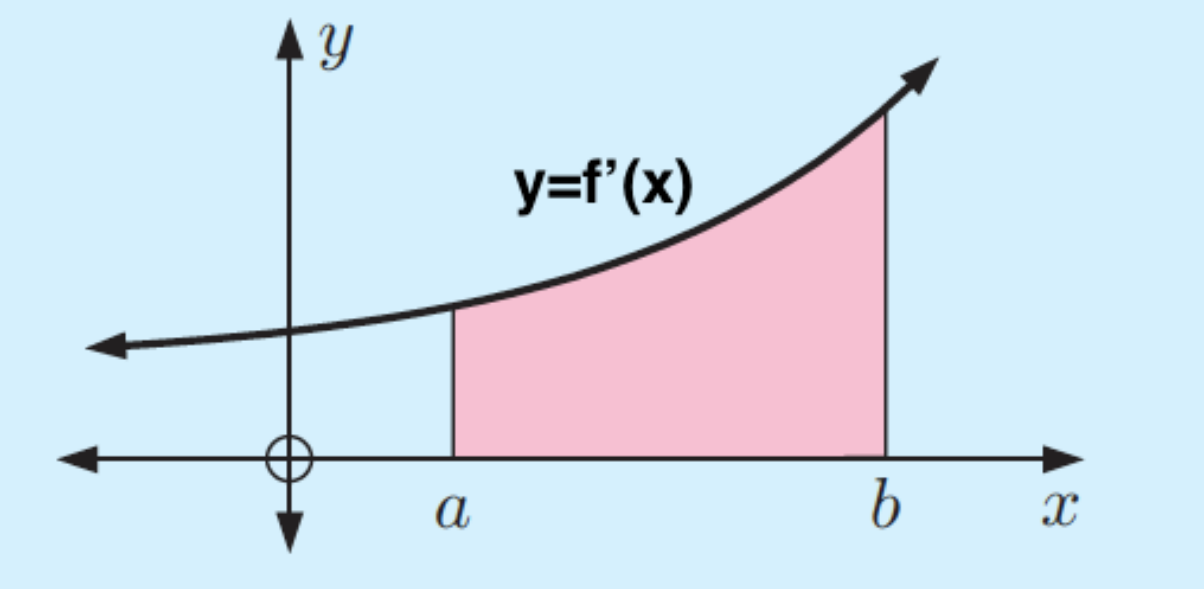
1. Find $\int 3x^2\,dx$.

2. Find $\int 5x^6 + 20\,dx$.

## Definite integral:

$$\int_a^b f'(x)\,dx$$

**Find the area under the curve.**




**The area below the curve between the line $x = b$ and $x = a$.**

$$Area = \int_a^b f'(x)dx\,, where\ b > a.$$

$$\int_a^b f'(x)\,dx$$
$$= [f(x)]_a^b$$
$$= f(b) - f(a)$$

## GDC skills

$$\int_{1}^{3} 2x + 1 \, dx$$

### Casio

OPTN → F4 CALC → F4 $\int dx$

### TI-84

Math → 9: fn Int(

### TI-nspire

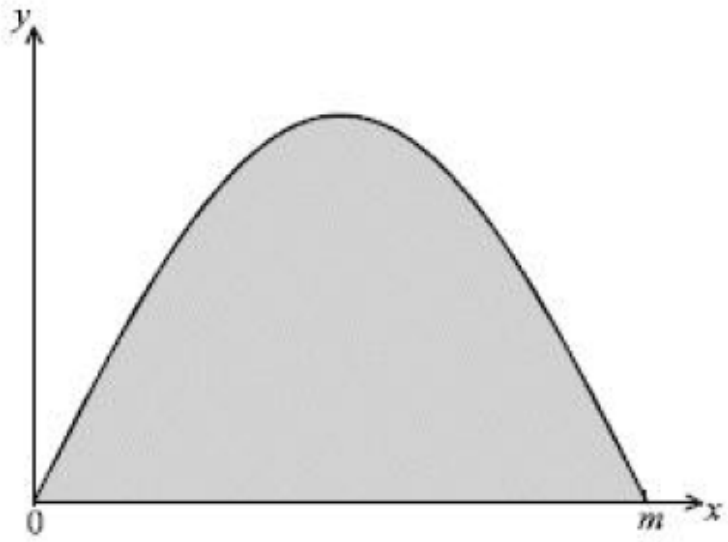
Menu → 4: Calculus → 2: Numerical integral

1. Find the area between x-axis and $f(x) = x^4$ from $x = 1$ to $x = 3$.

2. Find the area between x-axis and $f(x) = -x^2 + 4x + 2$ from $x = 0$ to $x = 2$.

## Exercise

1. The diagram below shows part of the graph of $y = -x^2 + 2x$.
The shaded region is between $x = 0$ $and$ $x = m$.

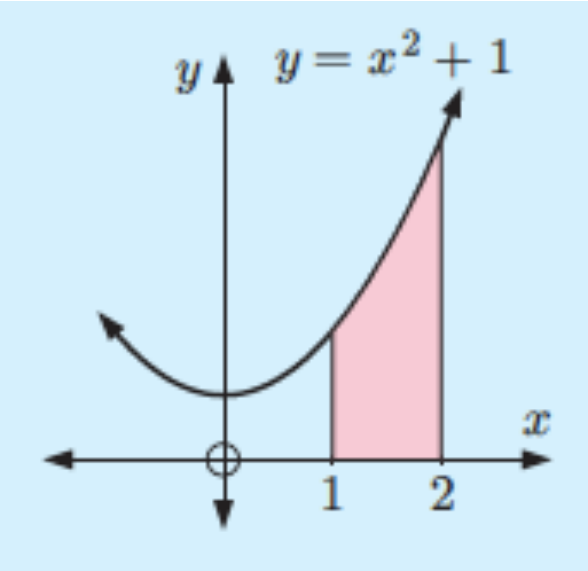



(a) Write down the value of m.

(b) Find the area of the shaded region.

2. Find the area of the region bounded by the curve $y = x^2$, the x-axis and $x = 2$.

3. Find the area of the region enclosed by $y = x^2 + 1$, the x-axis, $x = 1 \; and \; x = 2$.



4. The following diagram shows part of the graph of $f(x) = 3x + 1$.

   Find the area of the region enclosed by $f(x)$, the x-axis, $x = 0 \; and \; x = 2$.