Differentiation rules

Derivative

$$y' \qquad \frac{dy}{dx} \qquad f'(x)$$

Slope of the curve

Slope of the tangent line

Rate of change

Put power down, Power – 1

6.2	Derivative of x^n	$f(x)=x^n \Rightarrow f'(x)=nx^{n-1}$
	Derivative of $\sin x$	$f(x)=\sin x \Rightarrow f'(x)=\cos x$
	Derivative of $\cos x$	$f(x)=\cos x \Rightarrow f'(x)=-\sin x$
	Derivative of $\tan x$	$f(x)=\tan x \Rightarrow f'(x)=\frac{1}{\cos^2 x}$
	Derivative of e^x	$f(x)=e^x \Rightarrow f'(x)=e^x$
	Derivative of $\ln x$	$f(x)=\ln x \Rightarrow f'(x)=\frac{1}{x}$
	Chain rule	$y=g(u),\ u=f(x) \Rightarrow \frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}$
	Product rule	$y=uv \Rightarrow \frac{dy}{dx}=u\frac{dv}{dx}+v\frac{du}{dx}$
	Quotient rule	$y=\frac{u}{v} \Rightarrow \frac{dy}{dx}=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$

Simple differentiation

Find the derivative of the following functions.

1. $y = 4x$

2. $y = 5x^3$

3. $y = 10\sqrt{x}$

4. $y = 20$

Find the gradient of the curve

1. $y = 10x^3$ at $x = 2$

2. $y = 4x^2 - 5x + 2$ at $x = 1$

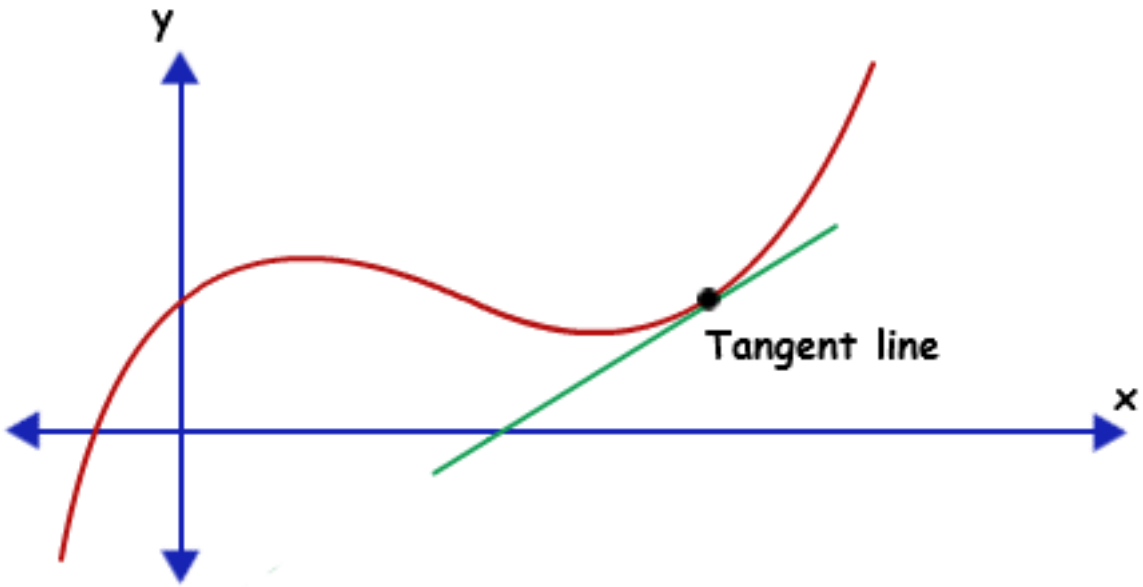
3. $y = \frac{3}{x^3} - 2x$ at $x = 3$

Equation of tangent and normal

Meaning of derivative $y', \frac{dy}{dx}, f'(x)$

Slope of the curve

Slope of the tangent line



Two steps to find the equation of straight line

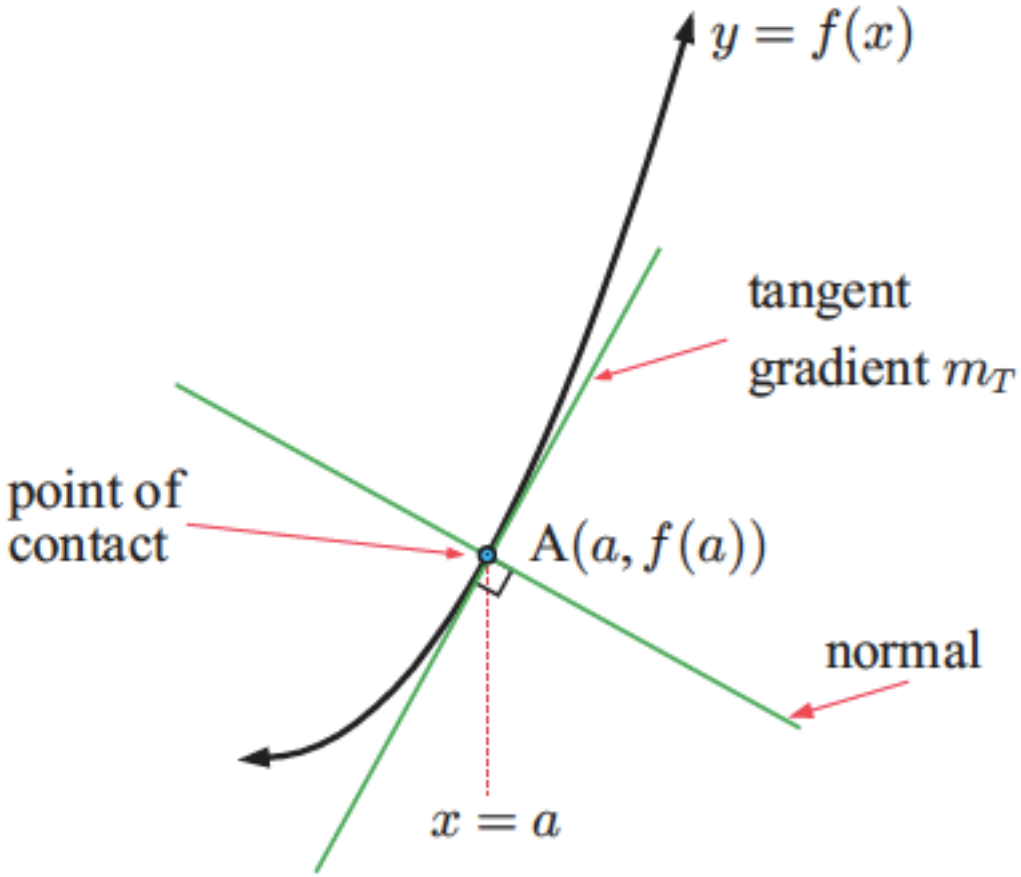
$$y = mx + c$$

1. Slope (m)
2. point (x, y)

1. Find the equation of the tangent to $f(x) = 2x^2 + 5$ at the point where $x = 2$.

2. Find the equation of the tangent to $f(x) = x^3 - 4x$ at (1, –3).

Equation of normal



$$\boldsymbol{m_T = f'(x)}$$

$$\boldsymbol{m_T \times m_N = -1}$$

$$\boldsymbol{m_N = \frac{-1}{m_T}}$$

1. Find the equation of the normal to $f(x) = 2x^2 - 10$ at the point where $x = -1$.

2. Find the equation of the normal to $f(x) = 5x^3 + \frac{2}{x^2} + 2$ at the point where $x = 2$.

Exercise

1. Consider the curve $y = 5x^3 - 3x$.

(a) Find $\frac{dy}{dx}$.

The curve has a tangent at the point $P(-1, -2)$.

(b) Find the gradient of this tangent at point P.

(c) Find the equation of this tangent. Give your answer in the form $y = mx + c$.

2. Consider the function $f(x) = \frac{x^4}{4}$.

(a) Find $f'(x)$.

(b) Find the gradient of the graph of f at $x = -\frac{1}{2}$.

(c) Find the x-coordinate of the point at which the **normal** to the graph of f has gradient $-\frac{1}{8}$.

3. A function f is given by $f(x) = 4x^3 + \frac{3}{x^2} - 3, x \neq 0$.

(a) Write down the derivative of f.

(b) Find the point on the graph of f at which the gradient of the tangent is equal to 6.

4. The equation of a curve is $y = \frac{1}{2}x^4 - \frac{3}{2}x^3 + 7$.

(a) Find $\frac{dy}{dx}$.

The gradient of the tangent to the curve at a point P is –10.

(b) Find the coordinate of P.