

Integration by substitution

No product rule and quotient rule for integration

Multiply

$$1. \int (x^2 + 3x)^3 (2x + 3) dx$$

$$\text{Let } u = x^2 + 3x$$

$$\frac{du}{dx} = 2x + 3$$

$$dx = \frac{du}{2x+3}$$

$$\int u^3 (2x + 3) dx$$

$$= \int u^3 (\cancel{2x+3}) \frac{du}{\cancel{2x+3}}$$

$$= \int u^3 du$$

$$= \frac{1}{4} u^4 + C$$

$$= \frac{1}{4} (x^2 + 3x)^4 + C$$

2. $\int 3x^2(x^3 - 2) dx$

Fraction

$$1. \int \frac{3x^2+2}{x^3+2x} dx$$

$$\text{Let } u = x^3 + 2x$$

$$\frac{du}{dx} = 3x^2 + 2$$

$$dx = \frac{du}{3x^2+2}$$

$$\int \frac{3x^2+2}{u} dx$$

$$= \int \frac{3x^2+2}{u} \times \frac{du}{3x^2+2}$$


$$= \int \frac{1}{u} du$$

$$= \ln u + C$$


$$= \ln x^3 + 2x + C$$

2. $\int \frac{4x^3 - 1}{x^4 - x} dx$

Paper 1

1.  Let $f'(x) = \sin^3(2x)\cos(2x)$.

Find $f(x)$, given that $f\left(\frac{\pi}{4}\right) = 1$.

2.  Let $g(x) = \frac{\ln x}{x}$.

(a) Find $g'(x)$.

(b) Find $\int g(x)dx$.
